Quantitative Results in Homogenization of Higher-Order Systems

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I will briefly introduce the quantitative homogenization of higher-order systems. These results are obtained under the guidance of prof. Zhongwei Shen during my visiting at University of Kentucky.

Consider the 2m-order Dirichlet elliptic systems,

$$\begin{cases} \mathcal{L}_\varepsilon u_\varepsilon = f & \text{in } \Omega, \\ Tr(D^\gamma u_\varepsilon) = g_\gamma & \text{on } \partial\Omega \quad \text{for } 0 \leq |\gamma| \leq m-1, \end{cases} \tag{0.1}$$

where

$$(\mathcal{L}_\varepsilon u_\varepsilon)_i = (-1)^m \sum_{|\alpha|=|\beta|=m} D^\alpha \Big(A_{ij}^{\alpha\beta}\Big(\frac{x}{\varepsilon}\Big) D^\beta u_{\varepsilon j}\Big), \quad 1 \leq i,j \leq n,$$

and A is 1-periodic. We established the $O(\varepsilon)$-convergence rate of u_ε to some function u_0 in $W_0^{m-1,q_0}(\Omega)$, i.e.,

$$\|u_\varepsilon - u_0\|_{W_0^{m-1,q_0}(\Omega) \leq C\varepsilon \|u_0\|_{H^{m+1}(\Omega)}},$$

where $q_0 = \frac{2d}{d-1}$. Moreover, we obtained kinds of uniform interior and boundary estimates for u_ε, such as Hölder estimates, $W^{m,p}$ estimates and large-scale $C^{m-1,1}$ estimates, under proper conditions. The parabolic systems of higher order and the elliptic systems with almost-periodic coefficients were also discussed in the other papers.